Hume's Principle and entitlement:

On the epistemology of the neo-Fregean programme*

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Abstract

Hume's Principle (HP) is the key principle in the neo-Fregean account of arithmetical knowledge developed and defended by Bob Hale and Crispin Wright. In discussing the epistemology of the neo-Fregean programme, Wright has proposed that acceptance of HP is warranted non-evidentially, as a matter of entitlement of cognitive project. The nature of entitlement of cognitive project remains a relatively unexplored matter. The aim of this paper is to shed some light on the notion, generally as well as specifically in relation to its intended application within the neo-Fregean programme. I start by providing some epistemological stage-setting and then introduce Wright's notion of entitlement of cognitive project. I proceed to investigate precisely how the notion is meant to apply in the context of the neo-Fregean programme. This is the first half of the paper. In the second half I raise and discuss three general issues pertaining to entitlement. Addressing these issues helps understand the nature of the epistemology adopted by the neo-Fregean.

1 Introduction

According to the neo-Fregean, an epistemological foundation can be provided for classical mathematics by appeal to abstraction principles. This proposal has been developed most fully—and been most widely discussed—in the case of arithmetic, where the abstraction principle favoured by the neo-Fregean is *Hume's Principle* (or 'HP', in short):

$$(\forall X)(\forall Y)(\sharp X = \sharp Y \leftrightarrow X \approx_{1-1} Y)$$

That is, for any concepts X and Y, the number of X's is the same as the number of Y's if and only if there is a 1-1 correspondence between X and Y.[1]

Given the central role played by Hume's Principle in the neo-Fregean enterprise, it is not surprising that the principle has received much critical attention in the literature.[2] The present paper adds to this discussion thread by scrutinizing Wright's proposal to the effect that acceptance of Hume's Principle is warranted non-evidentially, as a matter of entitlement.

2 Neo-Fregeanism and basic warrant

Before I move on to discuss entitlement, let me provide a bit of stage-setting to make clear why the good epistemic standing of Hume's Principle is of crucial importance to the neo-Fregean programme.

The neo-Fregean project is foundationalist in spirit. Arithmetical knowledge and warrant is to be accounted for by appeal to a set of basic principles, where by 'basic' is meant (at least) that the principles are non-inferentially warranted. For the neo-Fregean the set of basic principles—the foundation—is second-order logic supplemented by Hume's Principle. Part of the foundationalist package is a certain thesis about how warrant for belief in non-basic propositions is to be accounted for. Warrant to believe in non-basic propositions is acquired via inference from the basic principles, the idea

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[1] $X \approx_{1-1} Y$ is definable in pure second-order logic: $X \approx_{1-1} Y =_{df.} (\exists R)(\forall x)(Xx \rightarrow (\exists y)(Yy \wedge (\forall z)(Rxz \leftrightarrow z = y)) \wedge (Yx \rightarrow (\exists y)(Xy \wedge (\forall z)(Rzx \leftrightarrow z = y))))$.

[2] Critical voices include Boolos [3], Rayo [10], Shapiro and Weir [12], and Weir [14].

being that acceptance of these principles enjoys a positive epistemic standing that gets transmitted to beliefs in non-basic propositions inferred from them.

Against the background of this picture the following question becomes very pressing:

(BASIC WARRANT)

How is acceptance of basic principles warranted?

This is a pressing question, because the warrant to accept the basic principles is what is supposed to account for the warrant to believe non-basic propositions. Absent a warrant to accept the basic principles, things will not get started on the transmission picture. Why? For the simple reason there will be nothing to transmit. Given the specific set of basic principles relied on by the neo-Fregean, (BASIC WARRANT) raises the following two questions:

1. How is acceptance of the principles of second-order logic warranted?

2. How is acceptance of Hume's Principle warranted?

Each question is important and interesting. For the purposes of this paper, however, I leave aside the first question and focus exclusively on the second.[3] The neo-Fregean response to be explored below is that acceptance of Hume's Principle is warranted as a matter of entitlement. Wright's characterization of entitlement is presented in Section 3. Some interpretative and clarificatory remarks follow in Section 4, with a view to addressing the question precisely how the entitlement story is meant to apply in the context of the neo-Fregean programme. In Sections 5–7 I raise and discuss three general issues pertaining to entitlement. Addressing these issues helps understand the nature of the epistemology adopted by the neo-Fregean.

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[3] The philosophical standing of second-order logic has received considerable attention, as a topic in the debate concerning the neo-Fregean programme as well as a topic in its own right. For a contribution to the former debate, see Shapiro and Weir [12].

3   Entitlement of cognitive project

According to Wright, acceptance of Hume's Principle is warranted as a matter of entitlement. In his published work on neo-Fregeanism, Wright occasionally uses the term 'entitlement' explicitly to refer to a kind of warrant applicable to acceptance of Hume's Principle, but merely gives a gloss on the notion. The principle is supposed to be 'available without significant epistemological presupposition'[4] or 'innocent until proven guilty'[5]. These glosses point to the key feature of entitlement, *viz.* that it is a default species of warrant. Hume's Principle is warranted unless it is shown to be otherwise.

In more recent work on scepticism—and how best to respond to the challenge it poses—Wright characterizes entitlement in greater detail. The following characterization (or something like it—see fn. 6) is adopted:

> **Entitlement of cognitive project:** Acceptance of a proposition P is an entitlement of a given cognitive project if
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> (i) P is a *presupposition* of the project, i.e. if to doubt P in advance—or weaker: being open-minded about P—would rationally commit one to doubting (or being open-minded about) the significance or competence of the project;
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> (ii) we have no sufficient reason to believe that P is untrue; and
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> (iii) the attempt to justify P would involve further presuppositions in turn of no more secure a prior standing ... and so on without limit; so that someone pursuing the relevant enquiry who accepted that there is nevertheless an onus to justify P would implicitly undertake a commitment to an infinite regress of justificatory projects, each concerned to vindicate the presuppositions of its predecessors.[6]

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[4] Wright [16], p. 279 and Wright [17], p. 321.

[5] Wright [16], p. 282.

[6] Wright [19], pp. 191–192. Note that the characterization presented here is not identical to the one presented by Wright—yet, it is a fair interpretation of what he has in mind. Let me highlight the differences between Wright's own formulation and the one just given. First, Wright applies 'entitlement'

Let me say a little bit about how to understand the characterization.

We have cognitive projects pertaining to any subject-matter with respect to which we can undertake an investigation and *learn* things. That is, cognitive projects are projects whose successful execution amounts to a cognitive achievement. For instance, I might find myself wondering how big my suitcase is and learn that it is $19'' \times 14'' \times 8''$ when I measure it.

Let us turn to clause (i) and consider again the cognitive project of determining the dimensions of my suitcase. The proper functioning of my perceptual apparatus is a presupposition of this project. If I were to doubt (be open-minded about) about the proper functioning of my perceptual apparatus while undertaking the project, I would be rationally committed to likewise doubting (being open-minded about) the significance of the project itself. I would be so in the sense that I would be rationally committed to doubting (being open-minded about) whether I could learn the dimensions of my suitcase by executing the project. A presupposition of a cognitive project is thus an unavoidable commitment of that project: doubting (being open-minded about) it rationally commits one to doubting (being open-minded about) the significance of the project. Hence, the attitude held towards P must thus be one that excludes doubt (and open-mindedness) about the relevant presupposition.

Clause (ii) is a negative clause and captures the defining feature of entitlement—that it is a non-evidential species of warrant. An entitlement to accept P does not require the presence of positive evidence supporting P. Instead what is required is the absence of sufficient counter-evidence. We get a stark contrast when we compare my acceptance that my perceptual apparatus is functioning properly to my belief that my suitcase is $19'' \times 14'' \times 8''$. According to Wright, the former is warranted as a matter of

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to *propositions*. Entitlement is a species of warrant, and warrant is standardly conceived as something that applies to attitudes towards propositions rather than to propositions themselves. This is why the characterization given here is formulated in terms of acceptance. Several passages make it clear that Wright does see entitlement as something that applies to propositional attitudes rather than to propositions. Second, condition (i) differs from Wright's own explicit characterization, which is phrased only in terms of doubt. Doubt is a stronger attitude than open-mindedness in the sense that doubt that P amounts to a positive attitude towards its negation, while open-mindedness involves a positive attitude towards neither P nor its negation. In various places, Wright implicitly relies on the modified characterization of a presupposition given here (e.g., Wright [19], p. 193).

entitlement. As just highlighted, this does not require any investigative work to be done. Entitlement—or non-evidential warrant—is warrant by default. On the other hand, if warranted, my belief that my suitcase is $19'' \times 14'' \times 8''$ is so in the sense of being justified. Justification, on Wright's view, is evidential warrant—and, so, the warrant for the belief in question does require investigative work to be done.

Clause (iii) is a regress clause. If it is supposed that acceptance of P is entitled, the clause tells us that attempts to acquire an evidential warrant, or a justification, to accept P must lead to an infinite regress. This regress will consist of justificatory projects that involve some presupposition of no more secure a prior standing than P itself. That is, a presupposition that is either of less or equally secure a prior standing. An example given by Wright presents a regress that involves presuppositions of the same general kind.[7] To illustrate return once more to the project of figuring out the dimensions of my suitcase and suppose that I am entitled to accept that my perceptual apparatus is functioning properly relative to this project. Now, suppose furthermore that I grant that there is an onus on me to provide a justification for this entitled acceptance—that is, to provide evidence in its favour. How might I proceed? Perhaps I will go to the doctor to have my hearing, eye sight, and so forth checked. The doctor's findings might be communicated to me in a number of ways—verbally or in writing. The point now is that whichever way it is, acquiring the evidence that is supposed to deliver the justification draws in presuppositions of the very same kind as the one I set out to investigate. Whether I am told or read that my perceptual apparatus is functioning properly, I need to take it as a presupposition that my perceptual apparatus is functioning properly on that occasion. But that calls for a new investigation.

One more thing is worth noting about clause (iii). With a bit of unpacking, the clause can be taken to imply that no attempt to justify an entitlement P can improve its epistemic standing.[8] It seems plausible to suppose that the following minimizing principle holds for justification (i.e. evidential warrant): a justification for a proposition

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[7] Wright [19], p. 189.
[8] Here I have been helped by conversations with Carrie Jenkins.

cannot be any stronger than the weakest evidence supporting one of its presuppositions.[9]
Metaphorically, if we think of the infinite regress as a 'justificatory chain', the minimizing
principle says that justification for an entitlement P can be no stronger than the weakest
link in its justificatory chain. Since any attempt to justify P will involve presuppositions
of no more secure a prior standing, this means that attempts to justify P can bring no
improvement in its epistemic standing. Here it will be useful to return to the example
given above. As seen, the attempt to justify that my perceptual apparatus is functioning
properly—a presupposition of one of my cognitive projects—involves presuppositions of
the same general kind. These can be of no more secure prior standing than the initial
presupposition. They are, after all, of the same general kind. Thus, if the attempt to
justify P is supposed to improve on its epistemic standing, it must be concluded that it
is bound to fail.

The above remarks should suffice to introduce entitlement of cognitive project. As
already mentioned, Wright's proposal is that acceptance of Hume's Principle is warranted
as a matter of entitlement. Certain issues are crucial to understanding this proposal fully.
They are, nonetheless, issues that are left largely unaddressed by Wright. The objective
of the remainder of the paper is to discuss some of these issues and further to raise a few
critical points. As shall transpire, some of these issues or points pertain to entitlement
in general, while others concern entitlement as applied specifically in the context of the
neo-Fregean programme.

4   Hume's Principle and cognitive projects

Recall that entitlements are relative to specific cognitive projects. The first issue that I
will turn to is what kind of cognitive project acceptance of Hume's Principle might qual-
ify as an entitlement for on the neo-Fregean picture. This is a basic matter that Wright
does not say much about. However, the following passage does offer some pointers that

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[9] Wright [19], p. 191. Wright goes on to deny the principle in its full generality, the exceptions being presuppositions that are warranted non-evidentially. This is compatible with what has been said here since we are talking about justification, i.e. evidential warrant. Indeed, as far as I can tell Wright himself buys into the minimizing principle for justification.

put us in a position to table some qualified suggestions:

> I argued that we have an entitlement of project to take that concepts are in good standing, absent specific reason to think the contrary. But our concepts in general are in good standing only if standard means for explaining them are effective—establish them as such. That standard basic means, whether it proceeds by explicit stipulation of ground for and consequences of judgements which configure a given concept, or whether it proceeds by immersive explanation, as it were, of practices in which those grounds and consequences are acknowledged, is implicit definition. So we are entitled to take it that implicit definition is a generally effective means of conceptual innovation and hence that, in the absence of specific reasons for misgivings, it is productive of concepts in good standing in particular cases. The success of any particular—statement or process of—implicit definition in conveying a concept in good standing should be the default assumption.[10]

Applying the above line of thought to Hume's Principle—the neo-Fregean's favoured principle for implicit definition in the case of arithmetic—one kind of cognitive project would appear to be the following:

> **Cognitive project 1:** Hume's Principle is a presupposition of the neo-Fregean project of introducing the concept of number through implicit definition. This is so because the concept of number is a sortal concept. Any sortal concept F is characterized by having an associated criterion of identity by which Fs are individuated and distinguished from one another. This—serving as the criterion of identity—is what the right-hand side of Hume's Principle does for numbers on the neo-Fregean view. Doubt or open-mindedness about Hume's Principle rationally commits one to doubting that it is free from counterexamples or being open-minded about whether this is so—that is, whether there are cases in which two concepts stand in a 1-1 correspondence and yet their numbers are different, or cases in which numbers of concepts are the same although there is no 1-1 correspondence between their concepts. However, this kind of doubt or open-mindedness amounts to—or, in

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[10] Wright [21], Section IV.

any case, involves—doubt or open-mindedness about whether 1-1 correspondence is really the criterion of identity for numbers, and as such, whether the neo-Fregean way of introducing the concept of number succeeds in introducing a genuinely sortal concept at all.

On the neo-Fregean picture, it seems difficult to deny that Hume's Principle is significant in the way just suggested. Once it has been granted that the concept of number is a sortal concept and that Hume's Principle provides the criterion of identity for it, the above line of reasoning seems compelling.[11] The neo-Fregean should thus regard acceptance of Hume's Principle as a presupposition of the project of introducing the concept of number. Are there other kinds of cognitive project for which acceptance of Hume's Principle qualifies as a presupposition on the neo-Fregean view? It would seem so:

> **Cognitive project 2:** Acceptance of Hume's Principle is a presupposition of arithmetical projects. If one were to doubt or be open-minded about Hume's Principle, one would likewise be rationally committed to doubt or being open-minded about the significance of any arithmetical project.

Cognitive projects 1 and 2 are related. The reason why acceptance of Hume's Principle must be a presupposition of any arithmetical project whatsoever on the neo-Fregean view is traceable to its being a presupposition of attempts the project of introducing the concept of number. The significance of arithmetical projects is dependent on the availability the concept of number. After all, arithmetic is the theory of (natural) numbers, and the theory and projects within it will be no good unless its core concept—the concept of number—is available.

In sum, acceptance of Hume's Principle is a presupposition of a wide class of projects on the neo-Fregean view—including not just the project of introducing the concept of number, but also any arithmetical project whatsoever. This lines up rather nicely

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[11] It is clear from the work of Hale and Wright that they take the concept of number to be a sortal concept. See, e.g., Hale and Wright [8].

with entitlement as discussed in relation to scepticism about the empirical world.[12] In the empirical world case the sceptical attack is of a high degree of generality. The propositions attacked by the sceptic—e.g. that I am not a brain in a vat or that I am not being deceived by an omnipotent, evil demon—are presuppositions of more or less all of our cognitive projects concerning the empirical world. Doubting or being open-minded about whether I am a brain in a vat would certainly seem rationally to commit me to doubting or being open-minded about any project I may have pertaining to the empirical world—determining the dimensions of my suitcase, say. Since acceptance of Hume's Principle is a presupposition of arithmetical projects in general, entitlement in the neo-Fregean setting will likewise be of wide epistemic significance relative to the domain under discussion.

Before proceeding to the next section let me make a few remarks concerning the nature and scope of the considerations just offered. There is a variety of reasons why people might be reluctant to take on board the idea that Hume's Principle is a presupposition of the kinds of cognitive projects discussed above.[13] One might be a fictionalist and think that there is no such thing as numbers. For this reason one might hold that there is no such thing as arithmetical projects pertaining to numbers and, therefore, no such thing as being a presupposition of this kind of project. Or one might reject the story about sortal concepts and criteria of identity that features so prominently in the neo-Fregeanism championed by Hale and Wright, and plays a crucial role in spelling out why Hume's Principle qualifies as a presupposition of a wide class of cognitive projects on their view.

To make a start addressing this issue let me highlight a qualification that was made in connection with the earlier classification of Hume's Principle as a presupposition of Cognitive Projects 1 and 2. It was said that the principle counts as a presupposition relative to these projects *on the neo-Fregean view.* This qualification is important for present purposes. For the qualified status of Hume's Principle as a presupposition is

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[12] See Wright [19] for details.
[13] I am indebted to both of the anonymous referees for raising this issue.

compatible with there being all sorts of reasons why adherents or sympathizers of positions other than the neo-Fregean one might want to resist attributing Hume's Principle this kind of status.

What this points to is a distinctive feature of the entitlement proposal, whether applied in the context of the neo-Fregean programme, in connection with empirical world scepticism, or with respect to some other domain. The entitlement proposal, as an epistemology of basic principles or presuppositions, has a relativist dimension to it. Acceptances qualify as entitled relative to one's conception or view of the relevant domain or region of thought. This feature derives from the satisfaction of clauses (i) and (ii) being relative in this manner. (More on this in the sections to come.) Viewed in this way, the application of the entitlement proposal in our present setting contributes to the internal project of spelling out an epistemology of basic principles—Hume's Principle in particular—from the point of view of the neo-Fregean. This project is to be distinguished from the more ambitious project of showing that Hume's Principle must be regarded as an entitlement—and so, a presupposition—by everyone, whatever one's view on numbers happens to be. It is the former, more modest project that is the focus of this paper. It is also, I believe, the only of the two projects that can be successfully executed—one reason being the kind mentioned earlier. It is by no means clear that Hume's Principle will qualify as a presupposition of arithmetical projects on all views.

5 Reasons

The previous section concerned the issue how to understand clause (i) on the specific assumption that acceptance of Hume's Principle is entitled. This section concerns the question what it takes for something to be a reason. This is relevant to understanding clause (ii) ('we have no sufficient reason to believe P untrue'). This issue does not just arise specifically in the context of the neo-Fregean programme, but pertains to entitlement in general. Below I will offer a common-place distinction between two conceptions of reasons—one metaphysical, the other epistemic—that leads to substantially different

readings of clause (ii).

Metaphysical reasons.

On the first conception of reasons—the metaphysical conception—what reasons a subject S has are determined by the world. The world determines the truth-value of propositions, and the set of propositions—with their world-determined truth-values—constitutes the domain of reasons relevant to clause (ii). This conception of reasons may strike some as slightly odd. However, it does capture a good notion of a reason, at least in the sense that it lines up with certain ways of thinking or talking about reasons. Let me illustrate by way of example. Suppose that Susan is out for a drive in her car and that she has just started making a turn. Furthermore, suppose that unbeknownst to Susan a kid—who is completely oblivious to her surroundings—is playing in the street just around the corner. In this case it seems to make perfect sense to say that Susan has a reason to hit the brakes. What gives Susan a reason to hit the brakes here is the fact that there is a kid in the street, i.e. the world's being in a certain way.

The following feature of the metaphysical conception is worth highlighting: if it is assumed that every proposition P is either determinately true or determinately false, no false proposition Q can qualify as an entitlement. For if Q is false, it means that $\neg Q$ is true—and the truth of $\neg Q$ is a sufficient reason to believe Q false.[14]

Epistemic reasons.

According to the second conception of reasons—the epistemic conception—what reasons a subject has for adopting a certain doxastic attitude towards a proposition are given by the set of propositions that are within her epistemic, or cognitive reach. The propositions that are epistemically reachable by S are those propositions that S would hold some doxastic attitude towards were she to exercise her cognitive resources fully (where by 'cognitive resources' is meant S's current capacities, abilities, skills, methods, and

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[14] Here I am assuming that negation flips falsity to truth—as is the case for classical negation, for example.

procedures).

Note that, running along these lines, the domain of reasons that is relevant to clause (ii) will exclude propositions that S would never consider—e.g. those propositions she does not possess the conceptual resources to consider. Also, whether clause (ii) is satisfied is independent of S's *current* attitudes to the cognitively reachable propositions. What is relevant is what attitudes S would hold if she were to exercise her cognitive resources fully. Furthermore, it should be observed that a candidate entitled acceptance of P might be true, and yet be undermined by a false proposition which (i) would be believed by S after fully exercising her cognitive resources and (ii) suffices for thinking that P is untrue. On the other hand, it might be that a candidate entitled acceptance of Q is false, but satisfies clause (ii), because the domain of reasons contains no proposition whose truth suffices for believing Q false. This is a respect in which the epistemic conception of reasons differs from the metaphysical conception.

The metaphysical and epistemic conceptions of reasons make different domains of reasons relevant to clause (ii). These domains are strikingly different. On the metaphysical conception, the domain of reasons is determined by what *is* fact, while, on the epistemic conception, the domain of reasons is determined by what is *taken to be* fact. Below I suggest that a suitably modified version of the epistemic conception of reasons is the one that is relevant to understanding clause (ii).

6   Defeasibility

Wright intends entitled acceptances to be defeasible. They can be possessed at a given time, but lost later. This is due to clause (ii). It is meant to be understood in such a way that there might initially be no sufficient reason to think some proposition P untrue, and yet later such a reason might be present—in which case the satisfaction of clause (ii) is undermined. Unfortunately, neither of the two conceptions of reasons from the previous section supports a reading of clause (ii) that makes entitled acceptances defeasible. I

will first show why and then proceed to discuss if there is any way to accommodate defeasibility.

Let us turn first to the metaphysical conception of reasons. This conception commits one to the non-defeasibility of entitled acceptances, at least given two assumptions. Supposing that R is the region of thought that we are concerned with, the first assumption—bivalence—is this: every R-proposition is either determinately true or determinately false. The second assumption is that the truth-value of any R-proposition is stable, i.e. that it does not change over time.[15]

Consider a candidate entitled acceptance of P against the background of the metaphysical conception and these two assumptions. Recall that we get defeasibility if acceptance of P can be entitled at one point, but fail to be so at some later point. As highlighted earlier, only acceptance of true propositions can qualify as entitled on the metaphysical conception of reasons. So, suppose that P is true and its acceptance entitled—meaning, in particular, that clause (ii) is satisfied. By the stability assumption P will remain true. This means that it will remain the case that the world is such that nothing is sufficient to making P false. Since reasons for belief on the metaphysical conception are simply given by the facts—or the state of the world—this means that there is no sufficient reason to believe P untrue. That is to say, there is nothing to undermine the satisfaction of clause (ii) and render entitled acceptances defeasible.[16]

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[15] Although the remarks to be offered on defeasibility are general in nature, it is worth noting specifically that the two assumptions at hand are ones that should be granted in the context of a discussion of the neo-Fregean programme.

[16] Let me say a little bit about what work the bivalence and stability assumptions are doing here. I will do so by briefly discussing the domain of law, a domain where these two assumptions seem implausible.

Most systems of laws are incomplete. They typically fail to determine completely what is and what is not lawful. Not every legal proposition is determinately true or determinately false. Without determinacy it can be indeterminate whether or not there is any sufficient reason—understood in the metaphysical sense—to believe a given proposition P untrue. It might be that there are lots of propositions whose truth would imply the falsity of P and, so, would be sufficient reasons to believe P untrue. However, it may be that it is indeterminate whether these propositions are true. In that case the metaphysical reading of clause (ii) supports neither defeasibility nor non-defeasibility.

Let me now turn to the stability assumption. The law is what determines the truth-value of legal propositions. However, it does not do so stably. For the law is mind-dependent and is subject to change. Specific laws can be amended or otherwise modified, or dropped. New laws can be added to the existing law. Suppose that current law says that some legal proposition P is true. Well, the law-makers can change the law so P is not true. Today's crime might not continue to be a violation of the law, or the other way around. Without stability the metaphysical reading of clause (ii) could render entitled acceptances defeasible. Consider an entitled acceptance of P. Since acceptance of P is entitled, there is

Let us now consider the epistemic conception of reasons and see if this conception can support defeasibility. Suppose that S is entitled to accept P relative to some domain of reasons D_R and a fixed class of methods and procedures CM. Recall that D_R includes every proposition Q which S would believe if she were to investigate the question whether Q by fully exercising her capacities together with the methods or procedures of CM. Thus, the status of acceptance of P as an entitlement is determined by reference to the best qualified beliefs S can (counterfactually) come to hold given her current cognitive repertoire. In this sense there is no room for improvement or fluctuation with respect to what propositions are in D_R. For this reason entitled acceptance will be non-defeasible. Whether, in the epistemic sense, there is sufficient reason to believe a candidate entitlement P untrue is determined by D_R, and since D_R is stably fixed, so is the question whether or not clause (ii) is met.

In sum, we have that neither the metaphysical conception nor the epistemic conception of reasons supports a reading of clause (ii) that renders entitled acceptance defeasible. Or at least neither conception does so if we understand the conceptions in the way that we have thus far. I will now turn to the question whether there is any way to modify either conception so as to yield a reading of clause (ii) that supports the intended defeasibility of entitled acceptances. It turns out that there is. Below I present two ways in which this can be accomplished for the epistemic reading of the clause.

Assume that the epistemic conception of reasons is taken as background for understanding clause (ii). Then either of the following two modifications will support defeasibility:

(1) Allow clause (ii) to be satisfied relative to what propositions the subject would hold some attitude towards if she were to exercise her cognitive resources to a certain degree, but undermined if the subject were to exercise those very resources

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no sufficient reason to believe a proposition P untrue—meaning, on the metaphysical reading of clause (ii), that the world is in such a way that there is nothing sufficient to make P untrue. Yet, at a later time it might be that some proposition Q is made true by the law due to some change or addition, *and* that the truth of Q suffices for the falsity of P. In that case the satisfaction of clause (ii) would be undermined, and the entitled acceptance of P defeated.

more carefully or to a greater degree. (This degree, although greater, might fall short of fully exercising the resources in question.)

(2) Allow clause (ii) to be satisfied relative to what propositions the subject would hold some attitude towards if she were to exercise a fixed set of cognitive resources to a certain degree (full or not), but undermined if the subject were to exercise an extended, improved set of resources to the same degree.

Let me offer a few examples to shed light on (1) and (2).

I turn to (1) first. Suppose that Peter endorses Schume's Principle, and further, that on Peter's view Schume's Principle stands to arithmetic as Hume's Principle stands to arithmetic in the neo-Fregean programme. In particular, Schume's Principle is crucial to the introduction of the concept of number. Hence, just as Hume's Principle is a presupposition of the project of introducing the concept of number and arithmetical projects, Schume's Principle is a presupposition of these projects within the framework adopted by Peter. Clause (i) from the characterization of entitlement is thus satisfied. Suppose that clauses (ii) and (iii) are satisfied as well, i.e. that (ii) Peter does not hold any doxastic attitude that renders a sufficient reason to believe Schume's Principle untrue after exercising his cognitive resources carefully, and (iii) granting an onus to justify—i.e. acquire an evidential warrant for—Schume's Principle would commit him to undertaking an infinite regress of justificatory projects involving presuppositions of no more secure a prior standing than that very principle. Since clauses (i)-(iii) are all satisfied, it would seem that Peter is entitled to accept Schume's Principle.

However, now imagine that Peter receives a letter from John that leads him to realize—and believe—that Schume's Principle generates an inconsistency in the framework that he endorses. Although it is imagined that John's letter is what prompts Peter to arrive at this belief, we can further imagine that Peter's cognitive resources are such that he might have come to hold the belief if he had exercised his cognitive resources more carefully on his own—even if less than fully. In that case clause (ii) would go from

being satisfied to being undermined. But this is just to say that his entitlement to accept Schume's Principle is defeated. We thus see that modifying the conception of epistemic reasons along the lines of (1) enables the neo-Fregean to accommodate the defeasibility of entitled acceptance.

As for (2), suppose that acceptance of P qualifies as entitled relative to a set of propositions which subject S would hold a doxastic attitude towards if she were to exercise her *current* cognitive resources fully. It might be that there are certain propositions that S has never considered due to limitations in her current cognitive resources, but which she would hold some attitudes towards if her cognitive resources were to be improved in relevant respects by, say, the addition of new methods or procedures. It might also be that there are certain propositions that S holds some specific attitude towards, but which she would hold a different attitude towards provided that her cognitive resources were improved in relevant respects. In both of the cases just envisioned it might be that the domain of reasons determined by S's improved cognitive resources contains a sufficient reason for believing P untrue, and in this sense the entitlement to accept P can be said to have been defeated.

Again, let us consider an example. The example is a variation on the Schume's Principle case from above. Let the initial setup remain the same: Peter is entitled to accept Schume's Principle. However, unlike before, suppose that Peter would not believe anything sufficient to think Schume's Principle untrue if he were to exercise his current cognitive capacities fully. But now consider Paul who is able to derive a contradiction from Schume's Principle in the context of the system endorsed by Peter. Paul does so by using a new method he has developed—one that is not in Peter's cognitive repertoire at all. In this case there is an improved set of resources—one that includes Paul's new method—that would lead Peter to believe that Schume's Principle generates an inconsistency in his system. This belief would defeat Peter's entitlement to accept Schume's Principle. Hence, (2)—like (1)—offers a way in which the epistemic conception can be modified so as to render entitled acceptance defeasible.

7   The Generosity Problem

Let me turn to the third and final issue regarding entitlement that I will discuss here. We can get at the issue by looking at the following passage from Wright:

> This is a good result [that acceptance of anti-sceptical propositions is entitled], it goes without saying, only if it is selective—only if the entitlements generated turn out to be cornerstones of our actual ways of thinking about and investigating the world and do not extend to all manner of irrational and bizarre prejudices.[17]

The issue tabled here is that entitlement might be too generous a notion. Entitlements might not, as Wright worries, be sufficiently selective but 'extend to all manner of irrational and bizarre prejudices' rather than apply exclusively to 'cornerstones of our actual ways of thinking about and investigating the world'. Wright holds that the usual anti-sceptical propositions—that I'm not a brain in a vat, among others—fall in the latter category. However, what falls into the first category, the category of irrational and bizarre prejudices? Wright invites us to consider the following case:

> Suppose I postulate a tract of reality—it might be the realm of non-actual possible worlds as conceived by Lewis—which is spatio-temporally insulated from the domain of our usual empirical knowledge, and a special faculty—as it may be, our non-inferential 'modal intuition'—whose operation is supposed to allow us to gather knowledge about it.[18]

Is acceptance of the reliability of modal intuition entitled relative to the region of 'modalizing'? (Or, as Wright says, 'our primitive, non-inferential impressions of modal validity and invalidity.') According to Wright it might very well be:

> After all, I have—in the nature of the case, since I cannot compare its deliverances with the facts, independently ascertained—no reason to believe that it is unreliable (so long as its

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[17]   Wright [19], p. 195.
[18]   Wright [19], p. 196.

promptings are consistent); and any attempt to check on its functioning will presumably perforce to involve further modal intuition, 'of no more secure a prior standing.'[19]

Let me make an attempt to add a bit of detail. Accepting that modal intuition is reliable is a presupposition of modalizing, because modal intuition is supposed to be the only means by which the realm of possible worlds can be investigated. Accordingly, doubt about the reliability of modal intuition would rationally commit one to doubting the significance of whatever modal project one might engage in. So, clause (i) is satisfied.

Also, it would seem that there is no sufficient reason to think that modal intuition is unreliable—where this is understood in an epistemic sense. Considering *individual* pieces of modal information is not something that could lead us to believe high enough a proportion of them false. This is because, by assumption, we have no independent grip on the realm of possible worlds which can serve as an external standard against which we can measure the correctness of individual deliverances of modal intuition. Matters change somewhat if we consider sets of pieces of modal information. For in that case, as Wright indicates, one way that there could turn out to be sufficient reason to think modal intuition unreliable would be if we were to believe a high percentage of pieces of modal information to be inconsistent with each other, upon carefully or fully exercising modal intuition and other relevant capacities (such as a priori reflection). However, the point is that this does not seem likely. Clause (ii) thus also appears to be satisfied.

As for clause (iii), attempts to vindicate the reliability of modal intuition must give rise to an infinite regress of justificatory projects that involves some presupposition of no more secure a prior standing than the reliability of modal intuition itself. Now, to vindicate the reliability of modal intuition one would have to consider its deliverances and justify the claim that high enough a proportion of them are true. Yet, as emphasized earlier, modal intuition is the only means by which we can access the relevant realm. Hence, testing the reliability of modal intuition would inevitably bring on a need to presuppose the reliability of modal intuition—that is, a need to presuppose exactly

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[19]Wright [19], p. 196.

what was meant to be vindicated in the first place.

Clauses (i)-(iii) are all satisfied. As a result, accepting that modal intuition is reliable should qualify as being entitled. Indeed, generalizing on the case at hand, it would appear that we have an instance of entitlement whenever we postulate (i) a tract of reality R which does not interact in any way with domains accessible through our usual cognitive powers or methods, and (ii) a corresponding special faculty or method— the R-faculty or R-method—by means of which we can access the tract of reality in question. It is a presupposition of R-projects, as doubt about its reliability will rationally commit one to doubting the significance of the R-project. Likewise there is no sufficient (epistemic) reason to think it untrue that the R-faculty is reliable, and attempts to vindicate its reliability will lead to an infinite regress of justificatory projects involving some presupposition of no more secure a prior standing than the reliability of the R-faculty itself. Hence, acceptance of the reliability of the special R-faculty seems to be entitled for cognitive projects pertaining to the postulated tract of reality. The reasoning from the modal case can be modified to support this more general conclusion.

Now, let us return to the modalizing case for a moment. Does the entitlement to accept the reliability of modal intuition fall under the heading of 'irrational and bizarre prejudices' rather than being a cornerstone of our actual ways of thinking about the world? Arguably not. Modal reasoning is part of our actual thinking about the world. However, it would seem that *some* agents can be entitled in accepting what Wright would classify as irrational and bizarre prejudices. We get this result by applying the template extracted from the modal case.

Suppose that Mr. X believes in (i) a tract of reality, Pixie World, inhabited by pixies and completely isolated from domains which we can gain access to by our ordinary faculties or methods, and (ii) a special faculty, the Pixie Eye, detecting the whereabouts and doings of the pixies. Suppose, furthermore, that Mr. X's overall conception of the world—including (i) and (ii)—is consistent. When investigating Pixie World, is Mr. X entitled in accepting that the Pixie Eye is a reliable faculty? The case fits the generalized template presented above. This suggests that Mr. X's acceptance is indeed entitled—

and, surely, this entitlement is not part of our actual ways of thinking about the world. Rather, it must be deemed crazy or bizarre from our perspective.

Let us reflect on this case. It points us to (at least) two interesting features of entitlement that I would like to dwell on, if only briefly. First, we see that entitlement is sufficiently generous to apply quite widely—indeed, it would seem, to the Pixie World case as well as any other case that satisfies the template relied on to generate that particular case. Some might be inclined to regard this generosity as a problem of overgeneration. I see it not so much as a problem of overgeneration as something that serves to emphasize an important feature of entitled acceptance—namely, its relativity. This is the second interesting feature. There is a dimension of relativity built into each of the three conditions in the characterization of entitlement of cognitive project. For this reason entitled acceptance is relative, and different subjects may be entitled to accept different propositions. Mr. X may be entitled to accept that the Pixie Eye is reliable while this proposition is not among the ones that Wright—and other subjects like him (I count myself as one)—are entitled to accept.

Let me spell out precisely how each of the three entitlement clauses involves a dimension of relativity. Let us start with clause (i), the presupposition clause. Propositions are presuppositions relative to cognitive projects. Mr. X has cognitive projects pertaining to Pixie World and relative to these projects it is a presupposition that the Pixie Eye is a reliable capacity. Neo-Fregeans have a variety of projects pertaining to arithmetic and, as we have seen, relative to these projects Hume's Principle is a presupposition. As for clause (ii)—the 'no reason' clause—recall epistemic reasons are what determines whether the clause is satisfied or not. On both versions of the epistemic conception of reasons considered above, what qualifies as epistmic reasons for me may fail to qualify as such for you, or vice versa. This is because subjects may hold doxastic attitudes towards different propositions when they exercise their cognitive resources to some higher degree (the epistemic conception modified along the lines of (1) in Section 5), and because the class of resources and methods available to two different subjects may not be identical (the epistemic conception modified along the lines of (2) in Section 5). Lastly, turning

to clause (iii)—the regress clause—a dimension of relativity is manifests itself in the talk of 'attempts to justify', the reason being that attempts to justify do not float freely. They are made relative to the class of methods, capacities, and resources available to the subject.

As seen at the outset of the section, Wright brings up the potential worry that entitlement of cognitive project is too generous—that it applies to 'all manner of bizarre and irrational prejudices'. The considerations offered in this section suggest that subjects can be entitled to accept propositions that *we* would consider crazy or bizarre. Mr. X is entitled to accept that the Pixie Eye is reliable. In this way entitlement is quite generous. However, it is not sufficiently generous to grant *us* an entitlement to accept that the Pixie Eye is reliable. The underlying explanation of the wide applicability of entitlement is its relativity: provided that a subject's cognitive projects, world view, doxastic attitudes, and resources all line up the right way internally, she can be entitled to accept the basic presuppositions of her cognitive projects pertaining to some tract of reality. At the same time the relativity of entitled acceptance is what explains why entitlement does not overgenerate so as to entitle a subject to accept propositions that she considers—or would consider—bizarre or crazy. These propositions are precisely of the kind that do not line up nicely or do not fit with her cognitive projects, world view, and doxastic attitudes.

8 Conclusion

Hume's Principle plays a crucial role in the neo-Fregean programme. This paper has been concerned with the epistemology of Hume's Principle, especially Wright's proposal that acceptance of the principle is warranted non-evidentially, as a matter of entitlement. My hope is to have accomplished two things in this paper. First, I have tried to shed some light on the entitlement proposal specifically in the context of the neo-Fregean programme by discussing how, exactly, the proposal is meant to apply in that context. Second, I have tabled and addressed three fundamental issues pertaining to the notion

of entitlement. I hope that this is welcome contribution to the literature since the nature of entitlement is still a relatively unexplored matter.

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